**Research Article**

**Kinetic alfven waves in self-gravitating quantum dusty plasma with trapping effects**

*Benish Najeeb Khan and Ayub, M.*

Mirpur University of Science and Technology, Mirpur, AJ&K, University of Poonch Rawalakot AJ&K

---

**ARTICLE INFO**

*Article History:*
Received 25th April 2015
Received in revised form 16th May, 2015
Accepted 21st June, 2015
Published online 31st July, 2015

**Keywords:**

---

**ABSTRACT**
We have put a study of linear and nonlinear propagation of kinetic Alfven waves in dusty plasma. By making use of the Sagdeev potential approach, we have examined the amplitude and period of the solitary density waves. The self-gravitating effect is involved in terms of Jeans term. In linear treatment the Jeans effect play a vital role by in low magnetized plasmas, the wave dispersion decreases, when Jeans affect increases. The wave dispersion increases, when the dust density increases. In the solitary density structures, the amplitude and period of the wave decreases by increasing the Mach number when the other parameters are fixed.

**INTRODUCTION**

Kinetic Alfven waves (KAWs) propagating sideways to an exterior magnetic field in plasmas have been explored widely due to their application to the explanation and description of electromagnetic variances in space and laboratory plasmas. Information from the Freja spacecraft indicated that actions of sharp density thumps displaying KAW field aspects happen in the auroral area of the Earth’s ionosphere (Wahlund., 1994; Louarn., 1994). Hasegawa and Mima, (1976) and some other scientists (Hasegawa and Uberoi, 1982; Stenflo and Atmos, 1990; Wu and Wang, 1995; Wang., 1998) examined nonlinear KAWs in a low-β ($m_e/m_i << \beta << 1$, where $\beta = 4n_iT/eB_z^2$, $m_e$ and $m_i$ are the electron and ion masses, $n_i$ is the ion density, and $T$ is the active temperature) collisionless plasma. It was known that nonlinear localized wave arrangements can occur. Heavy, charged dust grains often appear in ionospheric and other plasmas. These huge charged dust grains change the properties of the already existing plasma. These can also lead new wave modes (de Angelis., 1989; Rao, 1990; Rao, 1995; Reddy, 1996). Numerous authors have calculated coherent structures in an unmagnetized dusty plasma (Rao., 1990; Popel., 1996; Das., 1996). It was concluded that very low frequency variations on the very long dust timescale can be excited. Chen Yinhua. Lu Wei and M. Y. Yu explored nonlinear dust KAWs (DKAWs), which are Alfven-like waves motivated by the polarization drift of the dusts and bending of the magnetic field lines (Chen Yinhua and Lu Wei, 2000). In this paper, we investigated the nonlinearities raised by trapping effect. The effect of trapped particles headed to a new type of nonlinearity; from this we obtained the solitary structures.

**Formulation**

We think about a collisionless plasma comprising of electrons, ions, and huge exceedingly charged dust particles. The consistent exterior magnetic field is specified by $\vec{B}_0 = B_0 \hat{e}_z$, where $\hat{e}_z$ is the unit vector along the z direction. Since the electrons are greatly speedier than the ions, the charge on the dust particles is generally negative.
We suppose $v_{ad} \gg v_{sd}$, where $v_{sd} = (Z_d T/m_d)^{1/2}$ and $v_{ad} = (B_0^2/4\pi n_0 m_d)^{1/2}$ are the dust acoustic and dust Alfven velocities, and $n_{do}, m_d, -Z_d e$ constant are the balanced dust density, dust mass and average dust charge, respectively. In terms of $\beta$ the above condition is $\beta \ll n_{i0}/n_{do} Z_d$, or $\beta_d \equiv 4\pi n_{do} T/\beta Z_d \ll 1/Z_d$. It’s the point to be noted that $\beta_d Z_d$ can be of the same order as $\beta$.

As the kinetic Alfven waves hold a transverse magnetic constituent, it is helpful to present for the electric field the longitudinal constituent $E_x = -\partial_x \phi$ and a diverse constituent $E_z = -\partial_z \psi (= -\partial_x \phi - c^{-2} \partial_t A_x)$, where $A_i$ is the vector potential parallel to the exterior field. The Maxwell equations lead to

$$\partial_t B_y = c \partial_y (\phi - \psi),$$

where $\psi_j$ is the longitudinal current. The densities of warm electrons and ions are specified by

$$n_e = n_0 (1 + \frac{\psi_j}{E_0})^{3/2},$$

$$n_i = n_0 \exp \left( -\frac{\psi_j}{T_i} \right),$$

where $T_e$ and $T_i$ are the electron and ion temperatures, respectively. The dust particles are cold and highly magnetized. In the drift estimation ($\partial_t \ll \Omega_d$, where $\Omega_d = Z_d e B / m_d c$ is the dust cyclotron frequency), the dust continuity equation can be written as

$$\partial_t n_d + \frac{1}{\beta_d} \sum_i \partial_i \left[ n_d \left( \partial_i \partial_x \phi - \frac{1}{\beta_d} \partial_i \partial_x \psi' \right) \right] = 0,$$

where $\psi'$ is the gravitational potential. We have ignored $v_{dz}$, we would fund to wave dispersion of higher order in $\beta_d$ and guide to a linear coupling with the dust acoustic waves. The network of Eqs. (1)-(5) is bolted by charge neutrality, current continuity and Poisson’s equations

$$n_e + Z_d n_d - n_i = 0,$$

$$\partial_z j_z = e \partial_t (n_e - n_i),$$

$$\nabla^2 \psi' = 4\pi G m_d n_d,$$

where we know that $\nabla j_z = -\partial_z j_z$, $j_{1z} \approx j_{dz}$, and $j_z \approx j_{ez} + j_{dx}$. Eqs. (1)-(8) give nonlinear DKAW motion in a dusty plasma. Terms which are of order higher than $\left( \partial_t / \Omega_d \right) \ll 1$ have been unnoticed.

**Dust Kinetic Alfven Waves**

By normalizing the Eqs. (1)-(8), we obtain

$$N_e = (1 + \Psi)^{3/2},$$

$$N_i = \exp \left( -\sigma \Psi \right),$$

$$\nabla^2 (\phi - \Psi) = \frac{1}{(1-\delta_e)} \nabla^2 (\delta_e N_e - N_i),$$

$$\partial_t N_d + \partial_x (N_d \partial_x \phi) - \frac{1}{\tau_d} \partial_l (N_d \partial_x \psi') = 0,$$

$$\delta_e N_e + Z_d \delta_d N_d - N_i = 0,$$

$$\delta^2 \psi' = \frac{1}{\epsilon_F} (4\pi G m_d^2 n_{do} N_d \rho_d^2),$$

where $\tau = \Omega_d t$, $\xi = x / \rho_{sd}$, $\xi = Z_d \omega_{pd} / c$, $N_j = n_j / n_{j0}$, $\rho_{sd} = (Z_d e B / m_d \Omega_d^2)^{1/2}$ which is the dust gyro-radius, $\omega_{pd} = \left( 4\pi n_{do} Z_d e^2 / m_d \right)^{1/2}$ which is the dust plasma frequency, $n_{i0} = \delta_d n_{do} \frac{\epsilon_F}{\epsilon_F} = \Psi$, $\sigma = \epsilon_F / \tau_l$. 

Now linearizing Eqs. (9)-(14) and considering that the perturbed variables are of the form \( \exp(i(k_1\xi + k_2\eta - \omega t)) \), we will get the (dimensionless) linear dispersion relation.

\[
\omega^2 = k_2^2 \left[ 1 - \frac{\omega_0^2}{\beta_d^2} + k_2^2 \left( \frac{1 - \delta_e}{2\epsilon_e + \omega} \right) \right] \quad \cdots \quad (15)
\]

This is the (dimensionless) linear dispersion relation for dust kinetic Alfvén waves.

**Localized Solutions**

We aspect for the quasistationary confined clarifications of Eqs.(9)-(14). Accordingly we present the moving coordinate \( \eta = \xi + \alpha \zeta - M_{Ad} \tau \), where \( \alpha \) and \( M_{Ad}/\alpha \) are direction cosines and the normalized wave speed by the sound speed. Considering a static configuration in the new setting, we have \( \partial_\xi = \partial_\eta, \partial_\zeta = \alpha \partial_\eta \), and \( \partial_\tau = -M_{Ad} \partial_\eta \). After performing some algebra, we get the following quadrature

\[
\frac{1}{2} \left( \frac{\partial \psi}{\partial \eta} \right)^2 + V(\psi) = 0 \quad \cdots \quad (16)
\]

where the Sagdeev potential is

\[
V(\psi) = \left( 1 + M_{Ad}^2 \right) \psi + 2 \left( \frac{M_{Ad}^2}{1 - \delta_e} + \frac{1}{\beta_d \delta_e M_{Ad}} \right) \int_0^\infty \left[ \delta_e \left( 1 + \psi \right)^{5/2} - 1 \right] \frac{1}{\sigma} (e^{-\sigma \psi} - 1) \quad \cdots \quad (17)
\]

here we have used the boundary conditions \( \eta \to \infty, \psi \to 0 \)

\[ \Phi \to 0, \partial_\eta^2 \Phi \to 0, \partial_\eta^2 \psi \to 0, \psi = 0 \]

for \( N_j \to 1 \) and effective dust-Alfven Mach number \( M_{Ad} = \frac{M_{Ad}}{\alpha} \). By applying the condition \( N_j = 1 \) at \( \eta \to \pm \infty \).

**RESULTS AND DISCUSSION**

In oblique part \((1 - \delta_e)\) of the dispersion (linear) relation presented in Eq. (15), both the dust density and dust charge affect it. By increasing the dust charge, the number density of electrons decreases. As a result, the wave dispersion increases with the increase in dust density.In our case the nonlinearities are raised by trapping effect. The effect of trapped particles headed to a new type of nonlinearity; from this we obtained the solitary structures. By the nonlinear treatment of our problem we got some graphical results under different conditions.

![Figure 1](image.png)

**Figure 1**. The Sagdeev potential \( V(\psi) \) for \( M_{Ad}^2 = 0.01 \) (thick) and \( M_{Ad}^2 = 0.015 \) (thin).

**Figure 1** represents the profile of the Sagdeev Potential when T, has a fixed value at 0.01 and the Mach number has different values i.e., 0.01 (thick) and 0.015 (thin).
Figure 2 (a) and (b) show the solitary density waves respectively, for the same parameters as in the case of Figure 1. In Figure 2 (a), the value of Mach number is 0.01, at this stage the amplitude of the wave is 0.08 and the period of the wave is 60. On the other hand, in Figure 2 (b), the value of Mach number is 0.015, at this stage, the amplitude of the wave is 0.03 and period of the wave is 40. From the solitary density wave representations we observe that as the value of Mach number increases the amplitude and period of the periodic density wave decreases.

Figure 3 gives the profile of the Sagdeev potential when $T_i$ and Mach number are fixed at 0.01 and 0.01 respectively and $Z_d$ has different values i.e., $10^4$ (thick) and $10^3$ (thin).

Figure 4 (a) and (b) show the solitary density waves respectively, for the same parameters as in the case of Figure 1. In Figure 4 (a), the value of Mach number is 0.01, at this stage the amplitude of the wave is 0.08 and the period of the wave is 60. On the other hand, in Figure 4 (b), the value of Mach number is 0.015, at this stage, the amplitude of the wave is 0.03 and period of the wave is 40. From the solitary density wave representations we observe that as the value of Mach number increases the amplitude and period of the periodic density wave decreases.
Figure 4 (a) and (b) describe the solitary dust waves respectively, for the same parameters as in case of Figure 3. In Figure 4 (a), the value of $Z_d$ is $10^4$, here the amplitude of the wave is greater as compared to that of in Figure 4 (b), where the value of $Z_d$ is $10^3$. In Figure 4 (a), the amplitude is at -0.08 and the period of the wave is 60. In Figure 4 (b), the amplitude is at -0.07 and the period of the wave is 20. From these observations, we conclude that by decreasing the value of $Z_d$, the amplitude and the period of the solitary density wave also decreases.

Conclusions

We have presented here a study of linear and nonlinear propagation of kinetic Alfvén waves in dusty plasma. Using the Sagdeev potential approach, we have investigated the amplitude and period of the solitary density waves. The self-gravitating effect is included in terms of Jeans. We see that in linear treatment the Jeans effect play a vital role by in low magnetized plasmas and the wave dispersion decreases, when Jeans affect increases. The wave dispersion increases, when the dust density increases. Moreover, we have seen that in the solitary density structures, the amplitude and period of the wave decreases by increasing the Mach number when the other parameters are fixed. Same behavior is seen when the dust charge is decreased while other parameters fixed.

REFERENCES


*******