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RESEARCH ARTICLE

APPLICATIONS OF LINEAR TRANSFORMATIONS

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ABSTRACT

In this paper, we applied linear transformations in the graph theory. Firstly, we constructed vector spaces of a finite number of elements over the field $\mathcal{F} = \{0,1\}$.Then we used them to construct linear transformations (in fact linear operators), defined on each of the vector spaces of the degrees of the vertices;$(D_V, +_V, \cdot_V)$; the vector space of the vertices;$(V, \oplus, \odot)$; and the vector space of the number of the paths between the vertex a and the other vertices;$(P_a, +_P, \cdot_P)$;.The last linear transformations is defined from V into D_V.

INTRODUCTION

Linear transformations appeared prominently in the analytic geometry of the seventeenth and eighteenth centuries. linear transformations also show up in projective geometry, founded in the seventeenth century and described analytically in the early nineteenth (Kleiner, I. 2007).

The idea of representing a linear substitution (i.e., a linear transformation) by the square array of its defining coefficients is already found in Gauss's treatment of the arithmetical theory of quadratic forms in 1801(Hawkins, T. 1974). Laguerre in France and Frobenius in Switzerland, further developed the consequences of the symbolical algebra of linear substitutions in a fashion similar to that taken by Cayley but without a knowledge of Cayley's memoir. Laguerre's work, which was published in 1867 in the journal of the EcolePolytechnique, ssuffered the same fate as Cayley's 1858 memoir(Hawkins, T. 1974).

Preliminaries

Definition 2.1(Faber, V. 2011) Undirected graph G is a nonempty set V called vertices together with a set E of 2-element subsets of V called edges.

Definition 2.2(Chartrand, G., & Zhang, P. 2019)Two vertices u and v in undirected graph$G$ are called adjacent in G if $\{u, v\}$ is an edge of G.

Definition 2.3 (Chartrand, G., & Zhang, P. 2019) The degree of a vertex v in undirected graph G is the number of vertices in G that are adjacent to v, and denoted by

$\deg(v)$ or $d(v)$.

Main Results(Applications)

3.1 Construction a linear operator onD_V

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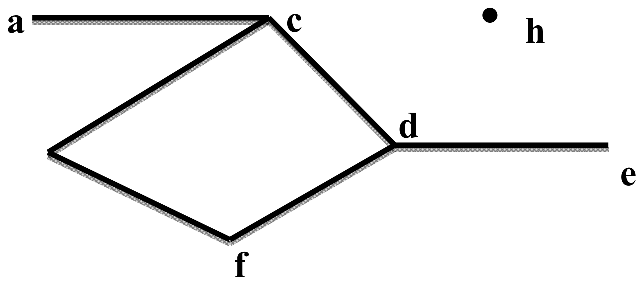

Figure (3.1) undirected G

Consider the following graph

$\deg(a) = 1$,
$\deg(b) = 2$,
$\deg(c) = 3$,
$\deg(d) = 3$,
$\deg(e) = 1$,
$\deg(f) = 2$,and
$\deg(h) = 0$.

We considered the elements of D_V as the degrees of vertices in the undirected graph G.
Then $D_V = \{0,1,2,3\}$. Define the operations $+_V$ *and* $._V$ on D_V by:

For $any\ x, y\ in\ D_V$

$$x +_V y = \begin{cases} \min\{x, y\}, & x \neq y \\ 0 & , x = y \end{cases}$$

And for $any\ x\ in\ D_V\ and\ r\ in\ \mathcal{F}$

$$r ._V x = \begin{cases} 0 & , r = 0 \\ x & , r = 1 \end{cases}$$

over the field $\mathcal{F} = \{0,1\}$ with operations $+_{\mathcal{F}}$ *and* $._{\mathcal{F}}$ which are defined by

$+_{\mathcal{F}}$	0	1
0	0	1
1	1	0

$._{\mathcal{F}}$	0	1
0	0	0
1	0	1

Then $(D_V, +_V, ._V)$ is a vector space over $\mathcal{F}$:

a- $Let\ x, y \in D_V, x +_V y = \min\{x, y\} \in\ D_V\ or\ x +_V y = 0 \in D_V\ , then\ x +_V y in\ D_V.\ Thus\ +_V\ is\ closed.$

b-$Let\ x, y \in D_V,\ x +_V y = \min\{x, y\} = y +_V x, or\ x +_V y = 0 = y +_V x, so\ +_V\ is\ cummutitve.$

c-$For\ any\ \ x, y, z \in D_V, x +_V (y +_V z) = x +_V \min\{y, z\} =\ (x +_V y) +_V z = \min\{x, y\} +_V z.$

d-$0 \in D_V\ such\ that\ 0 +_V x = \min\{0, x\} = 0, for\ any\ x \in D_V\ , so\ 0\ is\ the\ identity\ element.$

e- $For\ any\ x \in D_V\ , x +_V x = 0$. This means that every element x in $D_V$has an inverse which is x itself.

f-$Let\ x \in D_V\ and\ r \in \mathcal{F}\ then\ r ._V x = \{0, x\} \in D_V. Then\ ._V$ is$closed.$

g- $For\ any\ r \in \mathcal{F},\ \ x, y \in D_V\ then$

$$r ._V (x +_V y) = \begin{cases} 0 & , r = 0 \\ x +_V y & , r = 1 \end{cases}$$

$$r ._V x = \begin{cases} 0 & , r = 0 \\ x & , r = 1 \end{cases}$$

$$r ._V y = \begin{cases} 0 & , r = 0 \\ y & , r = 1 \end{cases}$$

$$\Rightarrow r._V\, x +_V r._V\, y = \begin{cases} 0 & , r = 0 \\ x +_V y & , r = 1 \end{cases}$$
$$= r\,._V\,(x +_V y)$$

h- $Let\ r, k \in \mathcal{F},\ x \in D_V\ then$

$$(r +_{\mathcal{F}} k)._V\, x = \begin{cases} 0 & , r +_{\mathcal{F}} k = 0\ (r = k) \\ x & , r +_{\mathcal{F}} k = 1 (r \neq k) \end{cases}$$

$$r._V\, x = \begin{cases} 0 & , r = 0 \\ x & , r = 1 \end{cases}$$
$$k._V\, x = \begin{cases} 0 & , k = 0 \\ x & , k = 1 \end{cases}$$

$$r._V\, x +_{\mathcal{F}} k._V\, x = \begin{cases} 0 & , r = k = 0 \\ x +_{\mathcal{F}} x & , r = k = 1 \\ x & , r \neq k \end{cases}$$

$$= \begin{cases} 0 & , r = k = 0 \\ 0 & , r = k = 1 \\ x & , r \neq k \end{cases}$$
$$= \begin{cases} 0 & , r = k \\ x & , r \neq k \end{cases} = (r +_{\mathcal{F}} k)._V\, x$$

i-$For\ any\ \ r, k \in \mathcal{F}\ and\ x \in D_V\ then$

$$r._V\,(k._V\, x) = \begin{cases} 0 & , r = 0 \\ k._V\, x\ , & r = 1 \end{cases}$$

$$= \begin{cases} 0 & , r = 0 \\ 0 & , r = 1, k = 0 \\ x & , r = k = 1 \end{cases}$$

$$= \begin{cases} x & , r = k = 1 \\ 0 & , o.w. \end{cases}$$

$$(\,r._{\mathcal{F}}\, k)._V\, x = \begin{cases} x & , r._{\mathcal{F}}\, k = 1 \\ 0 & , r._{\mathcal{F}}\, k = 0 \end{cases}$$
$$= \begin{cases} x & , r = k = 1 \\ 0 & , o.w. \end{cases} = r._V\,(k._V\, x)$$

j-$1._V\, x = x, for\ any\ x \in D_V$.

The function $T: D_V \to D_V$ is defined by $T(x) = x^{-1}$ is a linear transformation on D_V:
Let $x, y \in D_V\ and\ r \in \mathcal{F}$

i-$T(x +_V y) = \begin{cases} T(\min\{x, y\}) & , x \neq y \\ T(0) & , x = y \end{cases}$

$$= \begin{cases} (\min\{x, y\})^{-1} & , x \neq y \\ 0 & , x = y \end{cases}$$

$$= \begin{cases} \min\{x, y\} & , x \neq y \\ 0 & , x = y \end{cases}$$

$$T(x) +_V T(y) = x^{-1} +_V y^{-1}$$

$$= \begin{cases} \min\{x^{-1}, y^{-1}\} & , x^{-1} \neq y^{-1} \\ 0 & , x^{-1} = y^{-1} \end{cases}$$

$$= \begin{cases} (\min\{x, y\})^{-1} & , x \neq y \\ 0 & , x = y \end{cases}$$
$$= \begin{cases} \min\{x, y\} & , x \neq y \\ 0 & , x = y \end{cases}$$

So, $T(x +_V y) = T(x) +_V T(y)$.

ii- $T(r._V\, x) = \begin{cases} T(x) & ,r = 1 \\ T(0) & ,r = 0 \end{cases}$

$$= \begin{cases} x^{-1} & ,r = 1 \\ 0 & ,r = 0 \end{cases}$$

$$= \begin{cases} x^{-1} & ,r = 1 \\ 0 & ,r = 0 \end{cases}$$

$$r._V\, T(x) = r._V\, x^{-1} = \begin{cases} x^{-1} & ,r = 1 \\ 0 & ,r = 0 \end{cases}$$

$$= T(r._V\, x).$$

Hence, T is a linear transformation onD_V.

3.2Construction a linear operator on$V$

From the previous process, we can obtain a vector space $(V, \oplus, \odot)$ on the set of vertices $V = \{a, b, c, d, e, f, h\}$, with the operations $\oplus$ *and* $\odot$ which are defined by: For any $x, y \in V, and\ r \in \mathcal{F}$,

$$x \oplus y = \begin{cases} x & ,\deg(x) < \deg(y) \\ y & ,\deg(y) < \deg(x) \\ h & ,\deg(x) = \deg(y) \end{cases}$$

$$r \odot x = \begin{cases} h & ,r = 0 \\ x & ,r = 1 \end{cases}$$

The identity element is h, the inverse of each element is given by the following table

x	x^{-1}
a	e
b	f
c	d
d	c
e	a
f	b
h	h

The function $T: V \to V$ is defined by $T(x) = x^{-1}$ is a linear transformation on V:
Let $x, y \in V and\ r \in \mathcal{F}$,

i- $T(x \oplus y) = \begin{cases} T(x) & ,deg\ (x) < deg\ (y) \\ T(y) & ,deg\ (x) > deg\ (y) \\ h & ,de\ g(x) = deg\ (y) \end{cases}$

$$= \begin{cases} x^{-1} & ,deg\ (x) < deg\ (y) \\ y^{-1} & ,deg\ (x) > deg\ (y) \\ h & ,deg\ (x) = deg\ (y) \end{cases}$$

$$T(x) \oplus T(y) = x^{-1} +_V y^{-1}$$

$$= \begin{cases} x^{-1} & ,deg\ (x^{-1}) < deg\ (y^{-1}) \\ y^{-1} & ,deg\ (x^{-1}) > deg\ (y^{-1}) \\ h & ,deg\ (x^{-1}) = deg\ (y^{-1}) \end{cases}$$

$$= \begin{cases} x^{-1} & ,deg\ (x) < deg\ (y) \\ y^{-1} & ,deg\ (x) > deg\ (y) \\ h & ,deg\ (x) = deg\ (y) \end{cases}$$

So, $T(x \oplus y) = T(x) \oplus T(y)$.

ii- $T(r \odot x) = \begin{cases} T(x) & ,r = 1 \\ T(h) & ,r = 0 \end{cases}$

$$= \begin{cases} x^{-1} & ,r = 1 \\ h & ,r = 0 \end{cases}$$

$r \odot T(x) = r \odot x^{-1} = \begin{cases} x^{-1} & , r = 1 \\ h & , r = 0 \end{cases}$

$$= T(r \odot x).$$

Hence, T is a linear transformation.

3.3Construction a linear operator on P_a

Consider the same graph G:We considered the elements of P_a as the number of the paths between a and the other vertices in the undirected graph G. Then$P_a = \{0,1,2\}$.

Define the operations $+_P$ and $\cdot_P$ on P_a by:For $any\ x, y\ in\ P_a$

$$x +_P y = \begin{cases} \min\{x, y\}, & x \neq y \\ 0 & , x = y \end{cases}$$

And For $any\ x\ in\ P_a\ and\ r\ in\ \mathcal{F}$

$$r \cdot_P x = \begin{cases} 0 & , r = 0 \\ x & , r = 1 \end{cases}$$

over the field $\mathcal{F} = \{0,1\}$.Then $(P_a, +_P, \cdot_P)$ is a vector space over $\mathcal{F}$.

Define $T: P_a \rightarrow P_a$ from the vector space P_a to itself by $T(x) = x^{-1}$. We have been shown that T is a linear transformation: For $any\ x, y\ in\ P_a\ and\ r\ in\ \mathcal{F}$

i- $T(x +_P y) = \begin{cases} T(\min\{x, y\}) & , x \neq y \\ T(0) & , x = y \end{cases}$

$$= \begin{cases} (\min\{x, y\})^{-1} & , x \neq y \\ 0 & , x = y \end{cases}$$

$$= \begin{cases} \min\{x, y\} & , x \neq y \\ 0 & , x = y \end{cases}$$

$$T(x) +_P T(y) = x^{-1} +_P y^{-1} = \begin{cases} \min\{x^{-1}, y^{-1}\}, & x^{-1} \neq y^{-1} \\ 0 & , x^{-1} = y^{-1} \end{cases}$$

$$= \begin{cases} (\min\{x, y\})^{-1} & , x \neq y \\ 0 & , x = y \end{cases}$$

$$= \begin{cases} \min\{x, y\} & , x \neq y \\ 0 & , x = y \end{cases}$$

$$= T(x +_P y)$$

So,$T(x +_P y) = T(x) +_P T(y)$

ii- $T(r \cdot_P x) = \begin{cases} T(x) & , r = 1 \\ T(0) & , r = 0 \end{cases} = \begin{cases} x^{-1} & , r = 1 \\ 0 & , r = 0 \end{cases}$

$$r \cdot_P T(x) = r \cdot_P x^{-1} = \begin{cases} x^{-1} & , r = 1 \\ 0 & , r = 0 \end{cases}$$

So, $r \cdot_P T(x) = T(r \cdot_P x)$

Hence, T is a linear transformation on P_a.

3.4Construction a linear transformation from$V$toD_V

Consider the same graph: For any $x, y \in V, and\ r \in \mathcal{F}$

$$x \oplus y = \begin{cases} x & , \deg(x) < \deg(y) \\ y & , \deg(y) < \deg(x) \\ h & , \deg(x) = \deg(y) \end{cases}$$

And$r \odot x = \begin{cases} h & , r = 0 \\ x & , r = 1 \end{cases}$

For $any\ x, y\ in\ D_V\ and\ r\ in\ \mathcal{F}$

$$x +_V y = \begin{cases} \min\{x, y\}, & x \neq y \\ 0 & , x = y \end{cases}$$

$$\text{And} r._V\, x = \begin{cases} 0 & , r = 0 \\ x & , r = 1 \end{cases}$$

The function $T: V \rightarrow D_V$ which is defined by $T(x) = deg\ (x^{-1})$ is a linear transformation:Let $x, y\ \in V, and\ r \in \mathcal{F},$

i- $T(x \oplus y) = \begin{cases} T(x) & , deg\ (x) < deg\ (y) \\ T(y) & , deg\ (x) > deg\ (y) \\ T(h) & , deg\ (x) = deg\ (y) \end{cases}$

$$= \begin{cases} deg\ (x^{-1}) & , deg\ (x) < deg\ (y) \\ deg\ (y^{-1}) & , deg\ (x) > deg\ (y) \\ 0 & , deg\ (x) = deg\ (y) \end{cases}$$

$$T(x) +_V T(y) = deg\ (x^{-1}) +_V deg\ (y^{-1})$$

$$= \begin{cases} \min\{\deg\ (x^{-1}), \deg\ (y^{-1})\}, \deg\ (x^{-1}) \neq \deg\ (y^{-1}) \\ 0 \qquad , \deg\ (x^{-1}) = \deg\ (y^{-1}) \end{cases}$$

$$= \begin{cases} deg\ (x^{-1}) & , deg\ (x^{-1}) < deg\ (y^{-1}) \\ deg\ (y^{-1}) & , deg\ (x^{-1}) > deg\ (y^{-1}) \\ 0 & , deg\ (x^{-1}) = deg\ (y^{-1}) \end{cases}$$

$$= \begin{cases} x^{-1} & , deg\ (x) < deg\ (y) \\ y^{-1} & , deg\ (x) > deg\ (y) \\ 0 & , deg\ (x) = deg\ (y) \end{cases}$$

So, $T(x \oplus y) = T(x) +_V T(y).$

ii-$T(r \odot x) = \begin{cases} T(x) & , r = 1 \\ T(h) & , r = 0 \end{cases}$

$$= \begin{cases} deg\ (x^{-1} & , r = 1 \\ 0 & , r = 0 \end{cases}$$

$$r._V\, T(x) = r._V \deg(x^{-1})$$

$$= \begin{cases} deg\ (x^{-1}) & , r = 1 \\ 0 & , r = 0 \end{cases}$$

$$= T(r \odot x).$$

Hence, T is a linear transformation.

Conclusion

Vector spaces of a finite number of elements using concepts in graph theory have been defined. Such vector spaces as far as we know were defined for the first time.These vector spaces have been used to construct linear transformation. Conversely, We can construct un undirected graph from these linear transformations.

REFERENCES

Kleiner, I. 2007. *A history of abstractalgebra*. Springer Science & Business Media.

Hawkins, T. 1974, August. The theory of matrices in the 19th century. In *Proceedings of the international congress of mathematicians, Vancouver*, Vol. 2, pp. 561-570.

Faber, V. 2011. Review of chromatic graph theory by Gary Chartrand and Ping Zhang. *ACM SIGACT News*, *42*(3), 23-28.

Chartrand, G. and Zhang, P. 2019. Chromatic graph theory. CRC press.

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