

www.ijramr.com



International Journal of Recent Advances in Multidisciplinary Research Vol. 08, Issue 11, pp. 7265-7273, November, 2021

RESEARCH ARTICLE

INTEGRATION OF MAPLE MATHEMATICAL SOFTWARE IN THE TEACHING-LEARNING PROCESS OF TRIPLE INTEGRALS IN THE 2ND YEAR AT THIGHER INSTITUTE OF EDUCATION SCIENCES OF CABINDA

¹Catarina Eufémia Manuel and *Marcos João Púcuta²

¹Master in Mathematics Teaching at Higher Institute of Education Sciences of Cabinda ²Assistant Professor at Mathematics Department of Higher Institute of Education Sciences of Cabinda

ARTICLE INFO ABSTRACT Article History: This article presents the results of a research carried out in the 2nd year students at ISCED in Cabinda, on the teaching learning presents of triple integrals with the integration of Marle Mathematical

Received 20th August, 2021 Received in revised form 17th September, 2021 Accepted 24th October, 2021 Published online 30th November, 2021

Key words: Maple Mathematical Software, Triple Integrals and Teaching-Learning Process. This article presents the results of a research carried out in the 2^m year students at ISCED in Cabinda, on the teaching-learning process of triple integrals with the integration of Maple Mathematical Software. Triple integrals are one of the most difficult topics of Mathematical Analysis to master by most students of this Educational institution, as they do not have a deep mastery of immediate integrals rules, integration methods, Fubini's theorem, lack of inversion of integrationlimits mastery, identification of these limits from the graphical representation, graphic sketch of functions of one and several variables, transformation of Cartesian coordinates to cylindrical and spherical coordinates, calculation of triple integrals, non-use of Information and Communication Technologies, among others. These difficulties were identified through a questionnaire and interviews carried out on 5 teachers and 143 students, whose collected data were processed by the SPSS statistical software. The statistic data obtained allowed the elaboration of a didactic strategy to favor the teaching-learning process of the triple integrals with the integration of the Mathematical Software in order to contribute to the reduction of students' difficulties. It was validated by teachers Masters and Doctors in Mathematics.

INTRODUCTION

Human training has been one of the most priorities and challenges of today's society, which is the basis for solving social problems and the sustainable development of any country in the world. In Angola, educational policies that improve, favor and qualify the human training are adopted. The government has shown concern with the education system and significant increase inthe teaching and scientific research quality in Public and Private Universities, with the aim of providing solid knowledge to its citizens. In line with this purpose, it is necessary and emerging to improve the teaching-learning process of Mathematics and triple integrals as an essential tool in human life, because, when well mastered, it develops a logical reasoning applicable to the study of any science. Its teaching requires the use of appropriate strategies and methodologies to an enabling environment to stimulate motivate and streamline this process. Triple integrals are a cross-cutting topic with multiple applicability in real life as in Physics, Engineering, Chemistry, Biology, Economics, Statistics, among other areas. In their learning, students make many mistakes in applying the immediate integrals rules, in the use of integration methods, Fubini's theorem, inversion of integration limits, and identification of integration limits from graphical representation, graphical sketch of functions of various variables, transformation of Cartesian coordinates to cylindrical and spherical coordinates, calculation of triple integrals, among others. In this aspect, Cury (2003) carried out an experiment with Chemical Engineering students, having investigated the errors made in evaluations, in contents such as function graphs, calculation of limits, derivatives and integrals. Students arrive at the University without developing, in Basic and High School, the skills to interpret the Mathematical language, abstract, generalize and explore problems in different areas of knowledge.

Currently, in their teaching and learning, Information and Communication Technologies are used as a tool that streamlines this process. Its growth creates challenges in the Higher Education Institutions, but at the same time it is opening new opportunities for Institutions, as societies can only have economic and social success in the new world of information if they have a well-educated and trained population. (PINTO, 2002). There are several technological tools that help students understand and develop the skills of calculating triple integrals and graphical sketching of functions of one and several variables, such as: Matlab, Graph, GeoGebra, Maple, Maxima, Descartes, among others. Maple Mathematical Software is a main tool in the Mathematics Teaching, Computer Science, Engineering and understanding of the contents related to triple integrals, achieving a significant learning experience. Maple enables an innovation of appropriate strategies and methodologies and increases motivation, dynamism, numerical experimentation, impulses and students' interest in learning the concept of triple integral calculus, the graphic sketch of functions in bi- and three-dimensional space, change of Cartesian coordinates, in cylindrical and spherical coordinates favoring visualization (PUCUTA, 2020).

*Corresponding author: Marcos João Púcuta,

Assistant Professor at Mathematics Department of Higher Institute of Education Sciences of Cabinda.

Henriques (2006, p.267) highlights the impact of this Software on the graphical sketch of functions of more than one variable and states that the use of Maple Mathematical Software for the graphical representation of functions of several variables, sketching integration regions and, consequently, in the solution of multiple integrals it can become a means of relieving the student from this arduous task, but essential for them is the treatment of the problem. The research carried out in the 2nd year students of Mathematics Teaching at Higher Institute of Education Sciences (ISCED) in Cabinda, concerning the triple integrals, shows evidence that confirms the existence of the aforementioned difficulties. It aims to integrate the Maple Mathematical Software in the teaching-learning process of triple integrals to reduce the difficulties encountered in the 2nd year students of Mathematics Teaching course at ISCED-Cabinda. According to the data obtained from the questionnaires and diagnostic test carried out on these students, we can say that the percentage of unfit is greater than 80%.

In view of this reality, the following scientific problem has arisen:

How to favor the teaching-learning process of triple integrals with the integration of Maple Mathematical Software in the 2nd year of Mathematics Teaching at ISCED-Cabinda?

General considerations on the teaching-learning process of triple integrals at ISCED-Cabinda: Triple integrals are one of the fundamental topics of Mathematical Analysis II, taught in the Higher Education Institutions at National and International level, mainly at the graduation level of Educational Sciences and Faculties of Science and Engineering. This topic comes from the three-dimensional space (R^3), based on Analytical Geometry and Descriptive Geometry with emphasis on simple integrals, as they occupy a functional place in the study of functions of one and several variables and their respective graphical representations in bi (2D)space and three-dimensional (3D).

Triple integrals are a vast content of extreme importance to University students and require them to master simple integrals with the use of different integration methods (substitution, by parts, trigonometric substitutions, integration of trigonometric functions, functions that contain a trinomialsquare, rational functions by partial fractions, irrational functions and integration by differential binomials), conic sections, quadratic surfaces, double integrals, calculation of order 2 and 3 determinants, Jacobian matrix, inversion of integration limits, transformation of Cartesian coordinates into polar coordinates , spherical and cylindrical, graphic representation of functions of one and several variables, geometric reasoning, among other subtopics. Thus, one can consider Mathematical Analysis I as the first source and base of these integrals that complete the syllabus of Mathematical Analysis II. For a more consistent approach to these integrals, the program and some books on Mathematical Analysis used by professors were reviewed, such as: Piskunov (1980), Kudriávtsev (1989), Demodovitchi (1993), Jerónimo (1995), Leithold (2003), Pérez (2006), Stewart (2006) and De Araújo (2008). In Demodovitchi's book (1993), for example, the triple integrals appear in Chapter VII, Multiple and Curvilinear Integrals p.268. In this chapter, we begin with the study of double integrals in rectangular coordinates, exchange of variables in double integral, calculation of figure areas, calculation of body volumes, calculation of surface areas, application of double integral to mechanics and later to integrals triples. In the teaching-learning process of these integrals, there is a certain weakness in the analytical, algebraic and graphic representation. This articulation plays a preponderant role in its conceptualization and modeling. The definition of triple integral follows the steps of double integral, which in turn depends on the simple integral.

In this regard, Púcuta (2021) proposes the following:

A simple integral of a function f(x) is defined on a finite closed interval of the axisx. A double integral of a function f(x, y) is defined on a finite closed region R of the plane xy.

A triple integral of the function f(x, y, z) is defined in a closed solid region *D* of accordinate system *xyz*. You can define triple integrals as a definite integral of the function w = f(x, y, z) in the three-dimensional space in the *D* region of R³.

When dealing with the simplest case, when f is defined in a rectangular box

 $B = \{(x, y, z) | a \le x \le b, c \le y \le d, r \le z \le s\}$, the first step is to divide a rectangle into subboxes. Then divide the interval [a, b] into l subintervals $[x_{i-1}, x_i]$ of equal length Δx , divide [c, d] into m subintervals of length Δy and [r, s] in n subintervals of length Δz . The planes through the points of these subintervals parallel to the coordinate planes subdivide box B into lmn subboxes:

 $B_{ijk} = [x_{i-1}, x_i] \times [y_{j-1}, y_j] \times [z_{k-1}, z_k].$

Each subbox has volume $\Delta v = \Delta x \Delta y \Delta z$. In this way, the Riemann triple sum is formed.

 $\sum_{i=1}^{l} \sum_{j=1}^{m} \sum_{k=1}^{n} f(x_{ijk}, y_{ijk}, z_{ijk}) \Delta v,]$ where $(x_{ijk}, y_{ijk}, z_{ijk})$ is in B_{ijk} . Thus, the triple integral is defined as the Riemann sum limit, that is, the triple integral of f in box B is given by:

 $\iiint f(x, y, z) dv = \lim_{l \to \infty} \sum_{i=1}^{l} \sum_{j=1}^{m} \sum_{k=1}^{n} f(x_i, y_j, z_k) \Delta V, \text{ if the limit exists.}$

Where dv = dxdydz is called the triple integral volume differential.

$$\iiint f(x, y, z) dx dy dz = \lim_{lmn \to \infty} \sum_{i=1}^{l} \sum_{j=1}^{m} \sum_{k=1}^{n} f(x_i, y_j, z_k) \Delta V$$

If you choose a point as any point of each subbox, $A(x_i, y_i, z_k)$, you get an expression for triple integral:

$$\iiint_B f(x, y, z) dV = \lim_{l,m,n\to\infty} \sum_{i=1}^l \sum_{j=1}^m \sum_{k=1}^n f(x_i, y_j, z_k) \Delta V.$$

Stewart (2006, p.1009), in his book entitled "Calculation with early transcendent", presents Fubini's theorem or practical method for calculating a triple integral. This consists in expressing it through an iterated integral $B = [a, b] \times [c, d] \times [r, s]$.

If f is continuous in a rectangular box, then

$$\iiint_{B} f(x, y, z) dV = \int_{r}^{s} \int_{c}^{d} \int_{a}^{b} f(x, y, z) dx dy dz$$

For its calculation, use any of the permutations of the volume element and make sure that the order in which the integration limits are written is compatible. This theorem makes it possible to change the order of integration according to each one's convenience. The iterated integral indicates that it first integrates with respect to variable x (keeping y and z fixed). Then in relation to variable y (keeping x and z fixed) and to variable z(keeping x and y fixed). Furthermore, there are other possible forms of integration, all of which provide the same result. If you first integrate with respect to variabley, then z, and finallyx, then you have: $\iiint_B f(x, y, z) dV = \int_a^b \int_r^s \int_c^d f(y, z, x) dy dz dx$

Generally speaking, there are six possible ways to calculate triple integrals: dxdydz, dxdzdy, dydxdz, dydzdx, dzdxdy e dzdydx. The process of placing the integration limits in the triple integral and the transformation of Cartesian coordinates into cylindrical and spherical coordinates, graphic sketching is considered complicated by the students. Henriques, Attie and Farias (2007, p.78) reinforce this conception when they state that "in many cases, the graphic representation in three-dimensional space is difficult to do in the paper/pencil environment, which is only based on the two-dimensional plane (the paper)". Changing teaching strategies and methodologies enables students to understand the conversion of these limits, coordinates transformation and the calculation of triple integrals. Stewart (2006) states that to convert the triple integral in rectangular coordinates to cylindrical coordinates one writes:

 $x = r\cos\theta, y = r\sin\theta$ and z keep z as variable, where $r \ge 0, \theta \in (0, 2\pi), z \in R$, and dV is replaced by $rdzdrd\theta$. To transform the F = (x, y, z) of rectangular coordinates to spherical coordinates the following equations are used: $x = \rho \sin\theta \cos\theta, y = \rho \sin\phi \cos\theta e z = \rho \cos\phi$

For the region $E = \{(\rho, \theta, \varphi) | a \le \rho \le b; a \le \theta \le \beta, c \le \varphi \le d\}, a \ge 0, \beta - a \le 2\pi e d - c \le \pi$ one can find the Riemann sum in the spherical coordinates with the help of the mean value of the triple integrals. According to this author, the formula for converting the triple integral of rectangular coordinates into spherical coordinates, with appropriate integration limits and replacement of the elements of volume dV by $\rho^3 sen\varphi d\rho d\theta d\varphi$ is used when including the most general spherical regions: $E = \{(\rho, \theta, \varphi), | \alpha \le \theta \le \beta, c \le \varphi \le d, g_1(\theta, \varphi) \le \rho \le g_2(\theta, \varphi)\}$ in this formula, the integration limits for ρ are $g_1(\theta, \varphi)$ and $g_2(\theta, \varphi) e r \ge 0$,

$\theta \in [0, 2\pi], \varphi \in [0, \pi].$

Triple integrals in spherical coordinates are used when cone, sphere and ellipsoid surfaces form the boundaries of the integration region and this occurs when there is symmetry with respect to the origin.

The use of Maple Mathematical Software in the teaching-learning of triple integrals: Maple is a computational algebraic system initially made for commercial purposes. It constitutes a computer environment for computing algebraic and symbolic expressions, allowing the design of two- and three-dimensional graphics. Melo (2002, p. 42) states that mathematical software is considered to be a mathematical computing system that enables calculations to be carried out, both algebraic and symbolic. Allows the sketch of curves in the plane and surfaces in space, performs rotations for a better visualization of the constructed graph. Maple is an indispensable and useful tool for university students of Exact Science courses in disciplines that have a large number of tools that can be explored in various contents involving Mathematical calculations, such as: Linear Algebra, Mathematical Analysis, Analytical Geometry, Differential Geometry, Numerical Analysis, Ordinary Differential Equations, and Differential Equations with Partial Derivatives, among others. Henriques (2004, p.15) states that, given computational resources, "the student can visualize and analyze, in real time, the concepts inherent to a family of surfaces and objects that it can build, thus expanding the range of relationships between the objects involved in the situation under the study". Tall (1992, p. 7) values the visualization of one, two and three dimensions of other objects in Calculus, in addition to multiple integrals. According to Henriques (2006, p.267), the Maple Mathematical Software allows the graphical representation of functions of several variables, sketching integration regions and, consequently, in the solution of multiple integrals, it can become a way to relieve the student of this arduous task, but what is essential for him is the treatment of the problem. Students need a graphical representation of functions of various variables to understand and visualize the triple integrals concept. When you open the Maple 18 program, you will find a series of interfaces or basic aspects of the program as shown in figure 1.

Maple contains:

- Main menu, where the student has access to all the program's tools;
- Shortcut buttons for some of the most used features;

- Side menu or pallets, which allow saving the most used expressions by the student and, fundamentally, gives access to Mathematical operations already in the standard format. It also has Greek letters and the most common Math symbols;
- Shortcuts to create new document, new worksheet and help button;
- Buttons for accessing various functionalities, which direct the student to a text that explains the content that the student selected.



Figure 1. Window containing the main Maple software tool

To start executing the activity, click on, worksheet, or new document (New document), or simply click on the New Worksheet button present in the shortcuts. After creating this sheet, you have the workspace next. From this moment the Maple Mathematical Software will be ready to work interactively, providing you with keyboard instructions (according to a syntax) from the command line designated by the symbol ">" called prompt.

To save the activities, just click on the File menu and select save or save as.

The example below shows the calculation of the triple integral using this Mathematical Software.

Calculate $\iiint_{W} \frac{1}{x^2 + y^2 + z^2}$ where W is the region inside the cone

 $z = \sqrt{x^2 + y^2}$ bounded superiorly by the sphere $x^2 + y^2 + z^2 = 4$ and inferiorly by the sphere $x^2 + y^2 + z^2 = 1$.

Resolution

As it is a sphere and a cone, the resolution of this integral in Cartesian coordinates is quite complex. If we consider the point P = (x, y, z) any in W, it is observed that the radius OP intersects the solid surface (or the solid boundary) initially at r = 1 then at r = 2.

From the two equations of the sphere, the following system of equations is obtained:

$$\begin{cases} x^2 + y^2 + z^2 = 4 \\ x^2 + y^2 + z^2 = 1 \end{cases} \Rightarrow \begin{cases} \rho^2 = 4 \\ \rho^2 = 1 \end{cases} \Rightarrow \rho = 2 \text{and} \rho = 1$$

Therefore, $1 \leq \rho \leq 2$

From the expression = $\sqrt{x^2 + y^2}$, we obtain:

$$\rho \cos\varphi = \sqrt{\rho^2 \sin^2\varphi (\cos^2\theta + \sin^2\theta)} \operatorname{ascos}^2\theta + \sin^2\theta = 1$$

$$\rho cos \varphi = \sqrt{\rho^2 sen^2 \varphi}$$

 $\rho cos \varphi = \rho sen \varphi \Rightarrow cos \varphi = sen \varphi$ is $\varphi = \frac{\pi}{4} = 45^{\circ}$

The angle φ ranges from 0 (positive zaxis) to $\frac{\pi}{4}$ (cone wall); its variation is found in the projection of W on the xy plane: $0 \le \theta \le 2$.

Hence
$$W_{\rho\theta\varphi}$$
 is given by: $W_{\rho\theta\varphi}$:
$$\begin{cases} 1 \le \rho \le 2\\ 0 \le \varphi \le \frac{\pi}{4} \text{ and } dV = \rho^2 sen\varphi d\rho d\varphi d\theta\\ 0 \le \theta \le 2\pi \end{cases}$$

These are the integration limits that allow you to draw the graph from Maple and calculate the volume of the given integral. Henriques (2006, p. 27) warns that graphic representation in three-dimensional space is difficult in the pencil/paper environment. For this reason, the computer environment, especially Maple, provides undeniable help. It can allow, as an instrument, control over visual variables in the global interpretation of figural properties. To graph the function $f(x, y, z) = \frac{1}{x^2 + y^2 + z^2}$, enter the command plots [Implicitplot3d] preceded by square brackets and write the function given, the plans and close the square brackets preceded by a comma. In the same command, insert the intersection points, the style and other non-mandatory but optional commands, the axes, then close the curved parentheses followed by a semicolon and press enter;

As an option, click on the right side to animate, color or move the graphic, as shown in the figure below.



Figure 2. Graphical sketch of the function given with Maple Mathematical Software

By calculating the triple integral, you get:

Figure 3.Calculating the triple integral with Maple

Didactic strategy in the teaching-learning process of triple integrals: Strategy is a planning system applicable to an articulated set of actions to achieve a goal. According to Brousseau (2002, apud.Alves, 2011), the strategy is adopted by intuition or by the natural rejection of a previous strategy. A new strategy is, however, adopted as a result of experimentation. That is, accepted or rejected based on the student's assessment with respect to its effectiveness; such assessment may be implicit. Valle Lima (2012, p.157) defines strategy as a set of sequential and interrelated actions that, starting from an initial state (given by the diagnosis), reach the ideal state as a result of planning. The strategy has as components: the mission, objectives, actions, methods and procedures, resources, responsible people for the actions and time, in which they must be carried out, the implementation andevaluation forms. Remédios (2001), the didactic strategy is a system of teaching-learning actions that are linked to categories, as a complex of selected and organized actions and procedures, which, taking into account all the components of the process, allow to achieve the proposed educational goals. According to Nerelys (2003), the didactic strategy are research contributions, as they have the essential purpose of projecting the transformation process of the object under the study, from a real state to a desired state and define them as:" the way of planning and directing actions to achieve certain objectives. Hargreaves (1991) defines the didactic strategy as a set of procedures supported by teaching techniques, whose object is to lead the didactic action to a good thermal container, ie to reach the learning objectives. A good strategy implicitly acts towards where you want to go, what you want to achieve and the goals to be followed.

The diagram in figure 1 illustrates, in a global way, the didactic strategy elaborated to carry out the present research



Source: Elaborated by the author/2020

The table below shows the actions of steps in each strategystage.

Nº	STAGES	STEPS	ACTIONS
		Diagnosis	It was asked about the technological availability regarding the use of Maple Mathematical software in the teaching-learning process of triple integrals. It was asked about the students' preparation in the "triple integrals" content, as well as their computer skills.
1	Planning	Struture and awereness	Textbooks and Mathematical Analysis II program were analyzed, in particular the triple integrals topic. The Mathematical Analysis II program was structured in accordance to its objectives.
		Control	Results obtained from the pedagogical and technological diagnosis were processed.
		Selection and configuration	Methodological guidelines, exercise system and problems that lead to the triple integrals were introduced. A set of practical activities related to triple integrals were elaborated.
2	Elaboration	Verification	It was verified if the planned activities, the exercise system elaborated for the students are well defined and their objectives. It was verified if the use of Maple Mathematical Software responds to the conceived model.
		Control	The levels reached by the students in the fulfillment of the actions elaborated on the triple integrals were valued.
		Guidance	Each teacher, based on their mission and functions, guides the indications to be followed and provides guidance for the fulfillment of the triple integrals objectives.
		Execution	The main Maple 18 window and its commands were explored. Problems leading to triple integrals were calculated and solved using Maple Mathematical Software.
3	Implementation	Control	The fulfillment level of the proposed objectives was verified.
		Inquary	The results obtained by the students using Maple Mathematical Software were integrated.
4	Control and assessment	Valuetion	The results obtained in each stage were valued, as well as the transformation level achieved by the students with the use of Maple Mathematical Software.
		Reajustement	Adaptations and changes were made in actions, through positive aspects and students' difficulties detected throughout the research.

METHODOLOGICAL RESULTS

Population, Sample and Data Processing: in this research, 5 teachers and 143 students at ISCED-Cabinda, aged between 22 and over 45 years, were selected, making a sample composed of 148. A questionnaire and interviews were applied to both teachers and students in order to obtain their opinions on the teaching-learning process of the triple integrals. The statistical data obtained were processed by the SPSS statistical software.

The implementation of didactic strategy results: For its implementation in the 2^{nd} year students of Mathematics Teaching course, it was necessary to install the Maple Mathematical Software version 18 on their Laptops and in Computer Laboratory, then they were pedagogically and technologically prepared, obtaining the following results:

- Enrichment and revolution in the teaching-learning process of triple integrals, through the use of Maple Mathematical Software.
- Reduction of students' difficulties in solving exercises and problems that leads to triple integrals using Maple Mathematical Software.
- Teachers/students' communication establishment for the exchange of questions and answers using the Maple Mathematical Software.
- Satisfactory results obtained by students in the triple integrals topic, that is, 96% and increased motivation and interest in their learning.

The reliability degree of the results obtained in applying the questionnaire to teachers is 0.791 and that of students 0.781. These data demonstrate that there is internal consistency of them and the instrument used to collect information on the teaching-learning of triple integrals is valid.



Graph 1. Post - Test Results

The graph below shows the 2nd year students' results of the Post-Testat Higher Institute of Education Sciences of Cabinda. In this graph, it is observed that 72% of students had an Excellent result, 16% had Very Good, 8% Good and 4% of the students hada Poor result, which demonstrates that the integration of the Maple Software in the teaching-learning process of the triple integrals brings significant changes in terms of their academic performance, enables dynamism, numerical experimentation and visualization of integrals, promotes significant learning, especially with regard to the development of the capacity and ability to solve various mathematical problems, autonomy and critical students' thinking, provide opportunities for motivation and appropriation of this content.

Didactic strategy validation by the experts: For its validation the experts in Mathematical Analysis with a scientific degree of Doctor in Mathematics Teaching and extensive experience in the use of Information and Communication Technologies in the teaching of triple integrals, there were presented:

- Questionnaires addressed to teachers and 2^{nd} year students at Mathematics Teaching Department at the diagnosis stage.
- Table containing the results of the pre-test and post-test applied to students selected in the sample.
- General scheme of didactic strategy for the teaching-learning of triple integrals with the integration of Maple Mathematical Software.
- Questionnaire with indicators (design and implementation of didactic strategy, stages and steps of the didactic strategy, methodology used for the integration of Maple Mathematical Software, its relevance at ISCED-Cabinda, feasibility of applying the didactic strategy at ISCED-Cabinda) evaluated and the scale used in your assessment.

B- Bad, S- Sufficient, G- Good, VG- Very Good and E- Excellent.

According to these instruments, experts have validated it as Excellent.

DISCUSSION

The implementation of MAPLE Mathematical Software in the 2nd year of Mathematics Teaching course at ISCED-Cabinda had a satisfactory result, where more than 80% of students had a thorough mastery of the tools and commands used in the calculation of triple integrals and in the graphical representation of functions of several variables. These results show that technological availability constitutes a valuable tool for the teaching-learning process of Mathematical Analysis, thus facilitating the understanding of several Mathematical concepts such as: function of several variables, limit of functions of several variables, triple integral, among others and the graphic outline of these functions facilitates the visualization and interpretation of areas and volumes of figures in three-dimensional space. In this aspect, Tori (2010) recognizes the importance of inserting technological resources in teaching by stating that the application of resources and tools available on virtual platforms enables not only the management of available content, but also the teaching-learning process. Moran (2000, p.17), is another researcher who highlights its importance for the teacher, student and the whole teaching process, when stating that technologies allow a new enchantment at school, for the teachers and especially the students. The teaching-learning process of triple integrals, especially the Maple Software in an exploratory way, isa tool that enables dynamism, numerical experimentation and visualization of triple integrals.

CONCLUSION

Triple integrals constitute a relevant content of Mathematical Analysis II, and other disciplines such as Physics, Statistics, among other areas. Its teaching requires students to master the concepts of simple integral, double and triple integral, graphic representation of conic sections and quadratic surfaces (ellipsoid, hyperboloid, paraboloid, sphere, cylinder, etc.), transformation of Cartesian coordinates cylindrical and sespheric, calculation of triple integrals, among others. The Maple Mathematical Assistant is a valuable pedagogical tool for the study of triple integrals. Its use could facilitate the calculation of these integrals, understanding of their concept, graphic representation and interpretation of the functions of several variables, contributing to the reduction of students' difficulties in solving exercises and problems that lead to triple integrals. The results obtained in the implementation of the didactic strategy showed that the students overcame their difficulties on the calculation of triple integrals and graphical sketching of the functions of several variables.

REFERENCES

- 1. Alves, F. R. V. *Aplicações da sequência Fedathi na promoção do raciocínio intuitivo no cálculo a várias variáveis*. Tese. (Doutorado em Educação). Universidade Federal do Ceará, Fortaleza. 2011.
- 2. Cury, A. Dificuldades dos estudantes do curso de engenharia. 2003
- 3. De Araújo, G. V. Cálculo Integral e Diferencial. "um enfoque em suas aplicações". Brasil. 2008
- 4. Demidovitch, B. Problemas e Exercícios de Análise Matemática, Edição. Mir Moscovo. 1993
- 5. Hargreaves, D. J. Matemáticas y Didáctica. 1991
- 6. Henriques, A. L'enseignement et l'apprentissage des intégrales multiples: analyse didactique intégrant l'usage du logiciel Maple., 2006. p.27

- Henriques, A.; Attie J. P.; Farias L. M. S. *Referências teóricas da didática francesa: análise didática visando o estudo de integrais múltiplas com o auxílio do software Maple*. Revista Educação Matemática e Pesquisa, v. 9, n. 1, p. 51-81, 2007.
 Henriques, M. *Softwares Matemáticos. 2004, p.15*
- 9. Jerónimo, A. A. M. A. Elementos de Cálculo Diferencial e Integral. Lisboa, Portugal. 1995
- 10. Kudriávtsev, L.D. Problemas de Análisis Matemático. Editorial Mir Moscú, Rusia. 1989
- 11. Leithold, L. El Cálculo. 7^{ma} Edición. México: Oxford University Press. 2003
- 12. Melo, J. M. R. Conceito da integral: Uma Proposta Cmputacional para seu ensino e aprendizagem. São Paulo. 2002. p.42
- 13. Moran, J. Ensino e Aprendizagem inovadores com tecnologias auditivas e temáticas, campinas mediação pedagógica, 1ª Edição. São Paulo, Brasil. 2000, p.17
- 14. Nerelys, A. R. Estratégia metodológica para preparar a los docentes de Ciências Naturales. 2003
- 15. Pérez, F. J. G. Cálculo Diferencial e Integral. 2006
- 16. Pinto, A. C. Recordações verídicas e falsas: Avaliação de alguns factores. Psicologia, Educação e Cultura. 2002
- 17. Piskunov, N. Cálculo Diferencial e Integral. Tomo II. Editorial Mir Moscú. 5ta Edición. Moscú, Rusia. 1980
- 18. Púcuta, M. J. Integrais triplas. Exercícios resolvidos. 2021
- 19. Púcuta, M. J. Material de apoio aos estudantes de Mestrado em Ensino da Matemática. Cabinda-Angola, 2020
- 20. Remédios, J. M. Vias que contribuyen a transfornat los modos de actuación em el desempenõ profisional del docente de Secundaria Básica. 2001
- 21. Stewart, J. Cálculo. Volume II Pioneira, 2006. Pág.1009Valle Lima (2012, p.157)
- 22. Tori, R. Educação sem distância: as tecnologias interactivas na redução de distância de ensino e aprendizagem. São. Paulo: Senac, 2010.
- 23. Valle Lima. A. Algunas formas de salida de los resultados científicos y vías que se han utilizado para su obtención. Instituto Central de Ciencias Pedagógicas. Ciudad de la Habana. 2010, Pág.188-195.
