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RESEARCH ARTICLE

INTRODUCTION TO GRAPH THEORY AN APPROACH METHODOLOGY FROM THE PROBLEM OF ORIGIN PERSPECTIVE, ITS APPLICATIONS TO PROBLEM SOLVING, AT THE HIGHER INSTITUTE POLYTECHNIC OF CABINDA – ISPCAB

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ABSTRACT

The present research consists of proposing methodologies that aim to work on graph theory making use of problematic methods and heuristic principles of teaching-learning of Mathematics. It aims to develop a methodological proposal that contributes to the improvement of the teaching-learning process of Graph Theory and its applications in Higher Education, that is, in ISPCAB, based on the mental schemes presented by Biembengut and Hein. SPSS and R-Studio statistical software were used to extrapolate data obtained from the questionnaire applied to teachers and students and the pre-test and post-test that were applied to 55 students; An analytical approach was made to the theoretical foundations of the object of study, with an emphasis on teaching methods (which are the teacher's actions to organize teaching activities in order to promote student learning). This was followed by the characterization of the current state, where difficulties of various types were found, such as the lack of a clear, deep and objective introduction of the concepts related to the different types of graphs; difficulties in extracting the matrix, both adjacency and incidence, in a graph and difficulties in interpreting problems involving graph theory, by the way they are formulated. Then, we elaborate, present, indicate proposals and propose methodologies to be adopted to minimize, if not, overcome these difficulties through problematic teaching.

INTRODUCTION

Marciano, J. (2009) states that Mathematics constitutes the basic language of science and technology, occupies an important place in the development of culture and humanity, among other reasons, because it generates a model of thought, promotes the capacity for abstraction and is a powerful reality modeling tool. As early as the mid-14th century, Galileo argued that in order to understand the Universe, we must know the language in which it was written. And that language is mathematics. With the emergence of computer science and the development of computational elements (digital computer), the motivation to study mathematics from a practical point of view increased. The development of mathematical concepts that relate elements of discrete sets is quite recent, compared to the history of "continuous mathematics". Graph Theory, as a branch of Topology and which is strongly linked to Algebra and Matrix Theory, has a relatively recent origin (18th century) in the history of Mathematics. Developed in the 20th century, its importance has been imposed by its connections and applications in other sciences, as well as in other areas of Mathematics. It studies the relationships between objects in a given set. For this, structures $G(V, E)$called graphs are used, where V is a non-empty set of objects called vertices (or nodes) and E is a subset of unordered pairs of V. For NCTM (2000) & (1991), when dealing with the principles and norms for School Mathematics, referring to the norms of content, our research is strongly linked to *Algebra* and *Geometry* for using letters and their operations, as well as proportions and the measures of extension, especially the figures. Regarding process norms, our research was limited *to Problem Solving* and *Mathematical*

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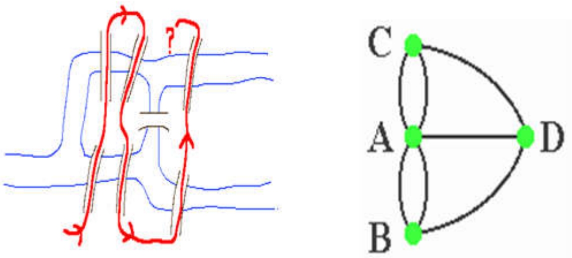
Connection due to the importance of the topic, its applications and, consequently, its connection with other sciences. Regardingskills, we emphasize *problem solving, communication* and *mathematical reasoning*. In the context in which this research takes place, *some shortcomings were detected* on the part of students and teachers regarding the teaching-learning process of graphs that are substantiated in: (i) the lack of a clear, deep and objective introduction of the concepts relating to the different types of graphs; (ii) little or no flexibility of thought on the part of students in the face of the relationship between Graphs and Set Theory; (iii) difficulties in extracting the matrix, both adjacency and incidence, in a graph; (iv) limitations or little knowledge of students on how to sketch a graph from an adjacency and/or incidence matrix; (v) difficulties on the part of teachers and students on the differentiation between paths, cycles (Hamilton) and chain (Euler) applied to problems. (vi) shortcomings in the methodological treatment of bipartite graphs; (vii) the lack of concern of teachers and students in solving problems involving Graph Theory; (viii) difficulties in interpreting problems involving graph theory, due to the way they are formulated; among others, in which we were predisposed to minimize or even solve the exposed problems. To address these shortcomings, we proposed the following scientific question:

How to favor the teaching-learning process on Graph Theory and its applications in Higher Education: A case study at the Institute Superior Polytechnicof Cabinda - ISPCAB?

The teaching-learning process of Graph Theory: Graph theory is introduced, implicitly, in primary education, in grade 7 and, in these classes, it is taught as geometry and all other elements that compose it. For Higher Education, in Educational Sciences, specifically in the Mathematics Teaching Course, graph theory is given in the 2nd year in the subject of Higher Geometry when dealing with Nikolai Lobachevsky's Geometry and in Operations Research when we seek to solve problems that involve paths and find the minimum longitude. On the other hand, in Computer Science, which is our focus, graph theory is given in the discipline of Discrete Mathematics and is used in several other disciplines such as Operations Research or Operations Research, Artificial Intelligence and its continuities, that is, I , II and II; in Physics, when dealing with electrical installations and Kirchhoff's laws and many other disciplines. In fact, it is important to emphasize that today the teaching of graph theory has been part of school life since primary education (Geometry), but it has been showing so many failures that it can be considered an element of exclusion, since most students cannot understand it. it.

In the city of Königsberg (now Kaliningrad), former capital of East Prussia, the River Pregel surrounds an island and separates the city into four zones that, in the 19th century, XVII were connected by seven bridges as in figure 1:

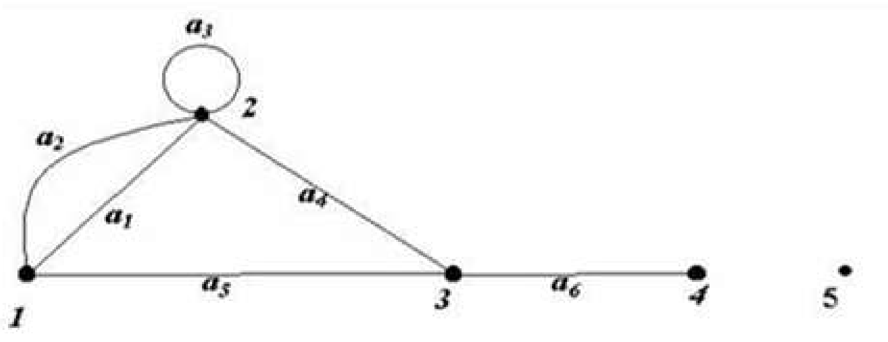


Source: Image adapted by the Author.

Figure 1. Idealized image of the Königsberg bridges

It is believed that this was one of the first examples of the use of graphs. The problem consists in starting from one of these regions and determining a route through the bridges according to which one can return to the starting region after crossing each bridge only once.

In the literature, the basic definitions of graph theory vary widely. Here are the nomenclatures and their used representations – A *directed graph* (also called digraph or quiver) consists of: A set V of *vertices* ; A set E of *edges* and Maps s , $t : E \to V$, where s (e) is the *source* and t (e) is the *target* of the directed edge e . An *undirected graph* (or simply graph) is given by A set V of *vertices* , A set E of *edges* and A function $w : E \to$ P(V) that associates to each edge a subset of two or one element of V , interpreted as the endpoints of the edge.



Source: Image adapted by the Author (ROSEN, 2009).

Figure 2. Graph of the combination of concepts.

A graph can be represented geometrically or algebraically. The algebraic way of representing a graph is through matrices, and this is how a graph can be identified in a computer system. The *adjacency matrix* $A = [a_{ij}]$ is an order matrix that algebraically represents a graph and is defined by:

Therefore $a_{ij} = 1$, when the vertices $v_i$and $v_j$are adjacent, and $a_{ij} = 0$otherwise.

$$a_{ij} = \begin{cases} 1 \ ; \text{ if and only if there is } (v_i ; v_j) \in E \\ 0 \ ; \text{Otherwise} \end{cases}$$

The value *matrix* represents valued graphs, where the numerical value of the edge can represent distance, capacity, flows, etc. Every single valued graph can be represented by its value matrix $W = [w_{ij}]_{m\times n}$, with its elements defined as follows:

$$w_{ij} = \begin{cases} \text{edge value if } (v_i ; v_j) \in E \\ 0 \text{ or } \infty \ ; \text{Otherwise} \end{cases}$$

In this way, if there is an edge joining two vertices, the value/cost of the edge is set as degree, if there is no such edge, the value set is zero or infinity. The *incidence matrix* an unoriented graph $G = G\,(V, E)$with n vertices on edges is denoted by $B = [b_{ij}\,]$and is a matrix $m \times n$defined as:

$$b_{ij} = \begin{cases} 1 \ ; \text{ if } v_i \text{ start vert ice of edge } e_i \\ 0 \ ; \text{Otherwise if } e_i \text{ a lasso} \end{cases}$$

If the graph $G$is oriented, then it can be defined as:

$$b_{ij} = \begin{cases} 1 \ ; \text{ if } v_i \text{ start vert ice of edge } e_j \\ -1 \ ; \text{ if } v_i \text{ start vert ice of edge } e_\text{j} \\ 0 \ ; \text{Otherwise if } e_i \text{ a lasso} \end{cases}$$

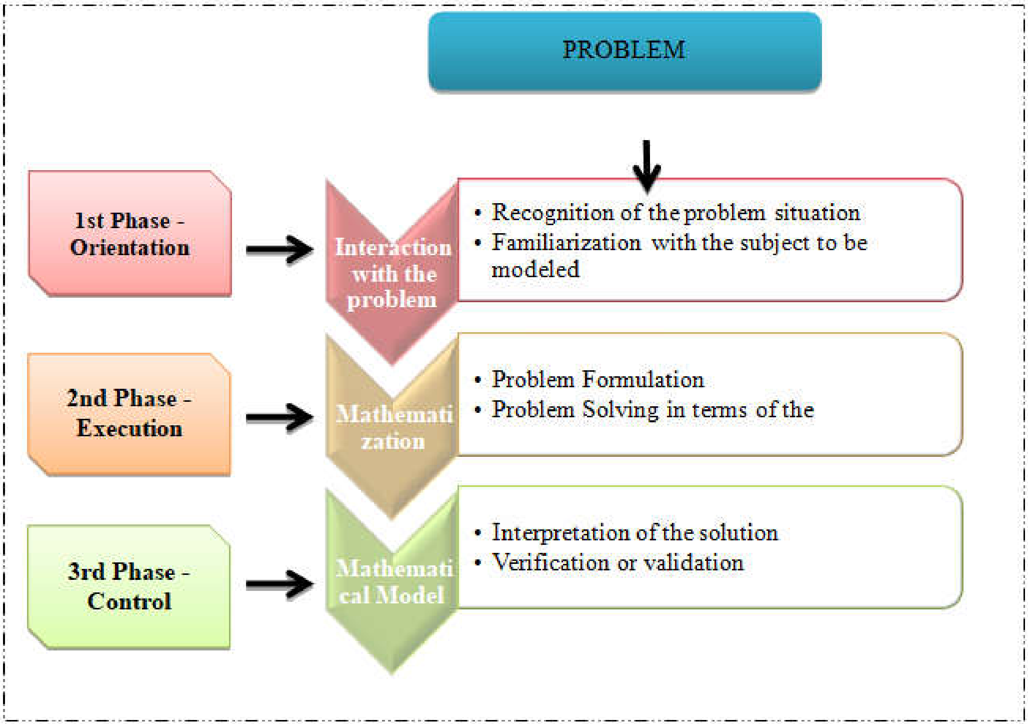
The Mathematics teacher must be, first of all, a mathematician teacher. Andrade (2013) defends that the teacher should present to the student all the ways of solving mathematical problems and leave it to each student to decide which is the best way to solve it. When the teacher uses this pedagogical method, he works with the student's logical reasoning, thus developing all his mathematical knowledge. When teaching content, the teacher must visualize the best ways to transmit knowledge, because within a classroom there are students with different mathematical skills and also different mathematical difficulties. To achieve this, it must be taken into account that the complexity of the mathematical content is considered from different points of view and its teaching and learning is carried out from a pluralist perspective where teachers are motivated to consider not only the different meanings of mathematics, as well as the diversity of its teaching.

MATERIALS AND METHODS

Mathematical Modeling and Heuristic Instruction in Problem Solving: Mathematical Modeling is a process that consists of translating problematic situations with genesis in everyday life or in other areas of knowledge, as long as they are meaningful to students, through the symbolic language of Mathematics, raising the need to use a set of symbols or mathematical relationships – Mathematical Model – to represent or organize the proposed problematic situation, in order to understand or solve it (de Almeida, Loiola Araújo, & Bisognin, 2011). Biembengut and Hein (2000, cited by Carvalho e Silva & Sant'Ana, 2002) refer that Mathematical Modeling "is the process that involves obtaining a model". Authors such as Ponte, Polya, Biembengut proposed procedures for solving problems. The latter groups and identifies these procedures in three steps, subdivided into six substeps. (i) Interaction with the problem: a) recognition of the problem situation and b) familiarization with the subject; (ii) Mathematization: a) problem formulation – hypothesis and b) problem solving in terms of the model; (iii) Mathematical Model: a) interpretation of the solution and b) validation. It should be noted that it is necessary to verify to what extent the model found satisfies the problematized situation. If the model does not meet the needs that generated it, the process must be resumed from the second stage, reorganizing it.

For Polya, "learning to think" is the great purpose of teaching. Learning must be active, modifying and process in consecutive phases. Thus, for this author, learning situations must be provided that arouse students' interest and in which they are challenged to discover results and establish relationships (POLYA, 1975).

Methodological actions to be adopted to improve the teaching-learning process of graph theory: They comprise the most varied possible situations of any problem that from the particular to the general point of view, there are several ways and models that satisfy a given problem, as we mentioned earlier. Thus, the conception of a problem is an initial phase by which an individual feels motivated to seek a solution. These actions contain methodological suggestions and learning activities involved in solving mathematical problems.



Source: Image adapted by the author.

Figure 3. Scheme of the development moments of any activity

1st Phase - Orientation: Interaction with the problem - this is the first step in solving a problem. In this phase, the situation to be studied will be outlined, and, to make it clearer, a research on the chosen subject must be carried out through books, newspapers, specialized magazines and data obtained from specialists in the area. The author in question suggests that one should start by reading the statement, in order to have an overview of the problem, as clearly as possible, and that the benefit of this is to understand the problem by relating it to a situation, familiarize yourself with it by highlighting the main objectives.

2nd Phase - Execution: Mathematization - this is the most complex and challenging phase, as it is in this phase that the problem-situation will be translated into Mathematical language, that is, it is here that a problem is formulated and written according to a model that lead to the solution from the current language to the mathematical language. Intuition, creativity and accumulated experience are essential elements at this stage.

3rd Phase - Control: Mathematical Model - the purpose of debugging is to verify the procedures used to reach the solution, trying to simplify them or seek other ways to solve the problem in a simpler way. And, abstraction aims to reflect on the process carried out, seeking to discover the essence of the problem and the method used to solve it in order to favor the transposition of the learning acquired in this work to solve other situations - problem.

Solving the problem by the model should only begin when you are sure that you are at a certain starting point and can supply all the minor details that may arise. The population for the present study will be considered finite, covering a total of 109 Students and 5 Professors of the Computer Engineering Course at the Institute Superior Polytechnic de Cabinda (ISPCAB) and who work with content related to Graph Theory where we work with all professors and, for students, we used simple random sampling, in which we obtained 52 students, among them 73.1% were male and 26.9% were female.

To obtain the results presented here, we applied a questionnaire and a pre-test and post-test. For the students, the applied questionnaire provided Cronbach's Alpha of 0.81 and, for the Teachers, 0.78, which shows a high consistency or simply a good internal consistency of the data and the measurement scale and, consequently, the validation of the instrument used to collect data and information for both cases.

RESULTS

Didactic structuring of mathematical problems involving graph theory, solved according to the application of the Biembengut and Hein strategy: we seek an example that characterizes a "model problem" and its resolution, schematized, following the proposal resolution strategy.

Problem: A surveyor is required to clearly, evidently and compactly place the existing neighborhoods between the Provinces of Angola. How would you help the surveyor using graph theory?

This is a problem that involves the extraction of the adjacency matrix in a graph.

1st Phase - Orientation: Interaction with the problem

Problem situation recognition: What does the problem say? What are the data? What are the conditions? What is the unknown? Is it possible to satisfy the conditions? Are they sufficient to determine the unknown? What kind of problem is this? What are the possible ways that the same exercise can be solved? Otherwise, let's see: a) Request for a surveyor; b) Neighborhood between provinces of Angola; c) Present the data in a clear, evident and compact way.

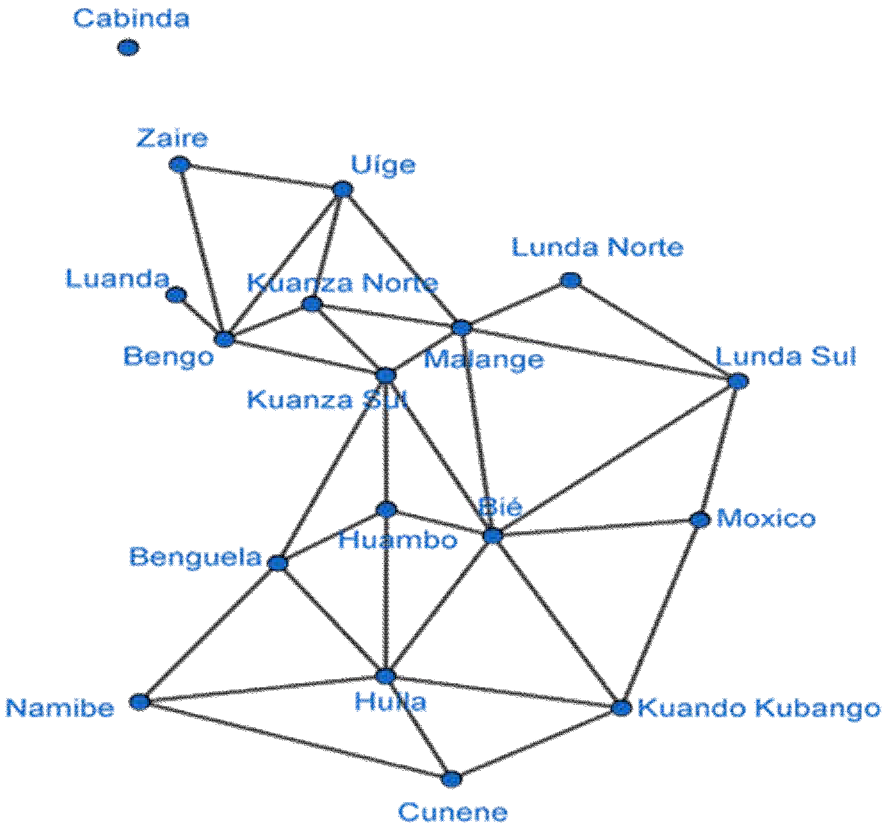
Familiarization with the subject to be modeled: In order to become familiar with the subject, we must encourage knowing every aspect of the situation, no matter how small: Can we represent the data by variables? What variables can be used? Do we know all provinces of Angola? Do we know the map of Angola? Are you aware of the type of content to use?. Due to the obvious evidence of the problem, we see that there is a need to obtain the map of Angola, trace the necessary edges or connections between neighboring provinces and develop an adjacency matrix in order to compact these neighbourhoods.

2nd Phase - Execution: Mathematization

Problem formulation - which operation will you use? Take a good look at the condition. What kind of problem is this? How to solve the graph problem? There is a problem here related to the one that has already been determined. Could you use it? Could you use the result? Could you use your method? Would you need to introduce some auxiliary element in order to be able to use it? Could you state the problem in another way? Could you present it in a different form again? Have you used all the data? Have you used all the condition? Have you considered the essential notions concerning the problem? Therefore, connections must be found between the data and the unknown. It is necessary to arrive at a model for the resolution.
Solving the problem in terms of the model - here, we must solve the model and check each step. Solve it step by step, check if it is done correctly, using the simple and flat graph to illustrate the neighborhoods of the provinces of Angola, in this sense, we have the following:

Figure 4. Map of Angola

Once the graph that illustrates the neighborhoods is finished, it is then extracted from the map in order to be able to observe these paths and neighborhoods with greater clarity. Then:



Source: Adapted by the Author

Figure 5. Map of Angola, Simple Graph

	Cab	Cun	Bo	Be	Bi	Hb	Hl	KN	KS	KK	Lda	LN	LS	Ml	Mx	Nb	Ug	Zr
Cabinda	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Cunene	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	1	0	0
Bengo	0	0	0	0	0	0	0	1	1	0	1	0	0	0	0	0	1	1
Benguela	0	0	0	0	0	1	1	0	1	0	0	0	0	0	0	1	0	0
Bié	0	0	0	0	0	1	1	0	1	1	0	0	1	1	1	0	0	0
Huambo	0	0	0	1	1	0	1	0	1	0	0	0	0	0	0	0	0	0
Huila	0	1	0	1	1	1	0	0	0	1	0	0	0	0	0	1	0	0
Kuanza Norte	0	0	1	0	0	0	0	0	1	0	0	0	0	1	0	0	1	0
KuanzaSul	0	0	1	1	1	1	0	1	0	0	0	0	0	1	0	0	0	0
Kuando Kubango	0	1	0	0	1	0	1	0	0	0	0	0	0	0	1	0	0	0
Luanda	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LundaNorte	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0
LundaSul	0	0	0	0	1	0	0	0	0	0	0	1	0	1	1	0	0	0
Malange	0	0	0	0	1	0	0	1	1	0	0	1	1	0	0	0	1	0
Moxico	0	0	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0	0
Namibe	0	1	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0
Uíge	0	0	1	0	0	0	0	1	0	0	0	0	0	1	0	0	0	1
Zaire	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0

After presenting the map of Angola in a simple graph, it appears that in this way it is possible to extract the adjacency matrix to observe the degree of each vertex (number of neighboring provinces). Once the matrix has been extracted, it is possible to verify the proximity of one Province to another.

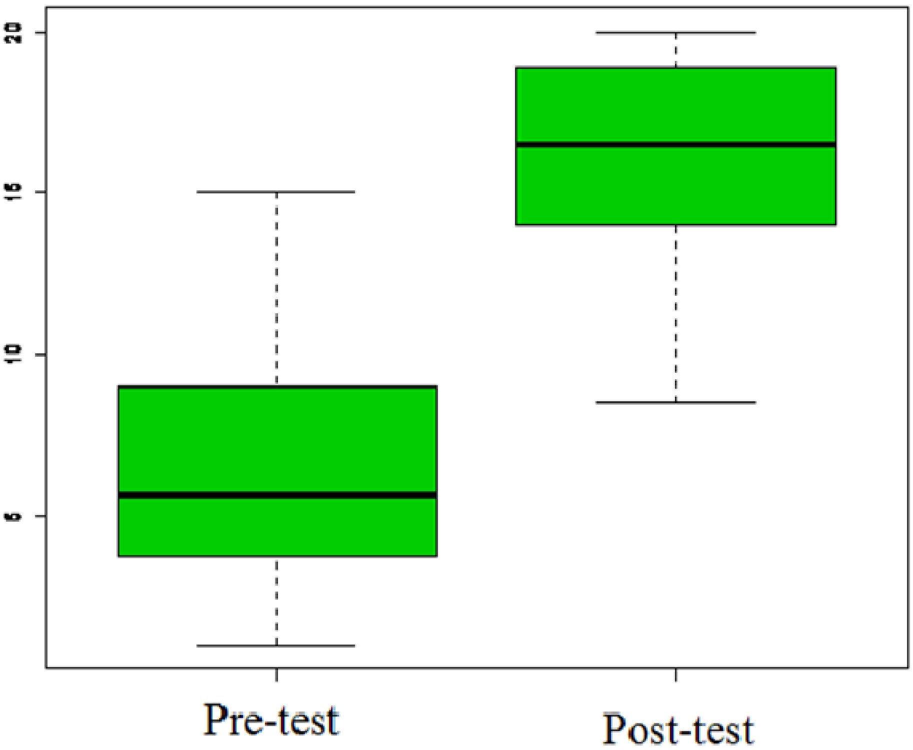
3rd Phase - Control: Mathematical Model

Interpretation of the solution - the obtained solution must be examined, verifying the results and the arguments used. Can you check the result? Can you check the reasoning? Or is it possible to check the argument? Can you get the result in different form? Or is it possible to arrive at the result by a different path? Is it possible to understand the argument in an array? Can you use the result or the method in some other problem? Or is it possible to use the result, method or procedure in some other problem? What is the essence of the problem and the solution method applied?

Verification or validation - to examine the result and other procedures, simply identify and/or select the province in which you want to know the amount of neighborhood you have, add the digits in each row or column and you will know if the province in highlighted how much neighborhood. It is also verified that we can know if a certain province has a neighborhood with another through the intersection of the row and column of each of the provinces. The highlighting note goes to the main diagonal of the matrix, which is completely zeroed, identity matrix, since a province cannot be a neighbor of its own. Note also that the Province of Bié is the province with the largest neighbourhood, 7 in total, and Cabinda is the one that does not have any neighbourhood, as it is a province geographically separated from the other provinces.

DISCUSSION

Implementation and validation of the methodological proposal: for the diagnosis of the problematic situation, a pre-test was applied that culminated in the results, as illustrated in the *box plot* . Taking into account the students' low performance, the *Methodological Proposal for the teaching-learning of Graph Theory was implemented: Methodological actions to be adopted to improve the teaching-learning process of graph theory* according to the target audience. Once the proposal was implemented, a post-test was applied again, which resulted in the results shown below. It is clear that after the application of the proposal and a second test, the students had an improvement in the results obtained in comparison to the results of the *pre-test*.



Source: Figure generated with the R-Studio environment – “Box-plot”

Figure 6. Pre-test and post-test results

In relation to the results obtained by the students in the *pre-test,* they present a minimum of 1 $value$and a maximum of 15 $value$e as an mean 6,69 $value$, while the *post-test results* present a minimum of8,5 $value$ and a maximum of 20 $value$e as an mean 15,82 $value$, which illustrates the success obtained with the presentation and application of the “ *Methodological Proposal* ” to the detriment of the traditional instruction carried out in the aforementioned institution. Validation of the proposal implemented through the evaluation of the Degree of Satisfaction: it was measured on a scale from 1 (not at all satisfied) to 10 (very satisfied). The questionnaire was the instrument used and applied to a sample of 55 students. *The satisfaction* variable was treated as quantitative in order to verify if its average is greater than 7. We applied the *t test* for a sample.

For a significance level, $\alpha = 0.05$, the following test hypotheses were established: *H_0 : The degree of satisfaction with the implementation of heuristic instruction in classes is equal to or less than 7 ($\mu \leq 7$). H_1 : The degree of satisfaction with the implementation of heuristic instruction in classes is greater than 7* ($\mu > 7$).

$$\frac{Sig}{2} = \frac{0{,}000}{2} < 0{,}00 \leq \alpha = 0{,}05 \quad \text{e} \quad t = 2{,}049 > 0$$

According to Table 1, as then the null hypothesis, H_0 is rejected (the alternative hypothesis, H_1 is accepted). Therefore, there is statistical evidence to say that the degree of satisfaction with the implementation of heuristic instruction in classes is higher than 7 ,

$$(t_{(54)} = 2{,}049 \; ; \; p - value < 0{,}001)$$

measured on a scale from 1 (not at all satisfied) to 10 (very satisfied). Thus, which reveals a high level of satisfaction, on the part of the students, in the implementation of *heuristic instruction* in their classes, since the average of satisfaction is significantly higher than 7, estimating that, with *95%* of confidence, it is in mean 8.04 points. The dispersion around the mean is relatively low (standard deviation of *3.751*) and student assessments are between 1 and 10, that is, varying by a maximum of 7 points.

Table 1. Statistics of a sample

	No	Mean	Standard deviation	Mean standard error
Degree of Satisfaction presented by the students	55	8.04	3.751	.506

Table 2. *t test* to assess the degree of satisfaction

One sample test					
Test Value = 7					
t	df	Sig. (2 ends)	Mean difference	95% Confidence Interval of the Difference	
				Bottom	Higher
2.049	54	.000	1.036	.02	2.05

Source: Frame generated with the *SPSS environment.*

CONCLUSION

We believe that this is a topic whose approach is not limited here, so we will not conclude the work, but rather weave some considerations that we think are relevant in the achievement of objectives related to our proposal. Even so, the results obtained in the implementation of this proposal of methodological actions showed us that: 1) Graph theory, despite being a relatively recent area of mathematics, is important due to its multidisciplinary connection with other disciplines; 2) Mathematical Modeling is a process that consists of translating problematic situations with genesis in everyday life or other areas of knowledge, as long as they are meaningful to students, through the symbolic language of Mathematics and applying it in the teaching-learning process. learning graph theory provides us with an added value; 3) The methodologies proposed here serve as a guide for solving, not only problems related to graph theory, but also several problems related to everyday life; 4) The system of exercises solved with the aforementioned methodology favor teachers and students, not only from higher education, but also from high school; 5) The methodological proposal presented here is a conceptual model that helps teachers to choose the best strategy for their learners' learning in relation to graph theory.

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